Abstracts of Lectures

Sarah E. Bailey

Non-Stationary Adic Transformations

We discuss a special family of Bratteli-Vershik systems (adic systems) for which the number of vertices per level increases at a constant rate. We compute that the dimension group of this family of systems is order isomorphic to the continuous functions modulo the continuous coboundaries. In addition we compute the dimension group explicitly for a subfamily of these systems.

Bohuslav Balcar

Dynamical systems and Boolean algebras

We deal with:

(i) Dynamics on extremally disconnected compact spaces.

(ii) Structure properties of universal minimal dynamical systems.

(iii) The multiple Birghoff recurrence theorem.

Francisco Balibrea Gallego

On the Periodicity Structure of Delayed Systems of the Form $x_{n+k} = f(x_n)$ with $k \geq 2$ in some Low-dimensional Spaces

We are dealing with delayed dynamical systems $(\mathbb{X}, f)$ where $\mathbb{X}$ is a low-dimension state space and $f \in C(\mathbb{X}, \mathbb{X})$ verifies the equation

$$x_{n+k} = f(x_n)$$

with $k \geq 2$. There are many examples of such type of behavior when $\mathbb{X} = \mathbb{I} = [0, 1]$ coming from Population Dynamics.

For such systems it is possible to find a frame of forcing relations among the periods that the continuous map f has similar to those stated in Sharkovskys ordering. When $\mathbb{X} = \mathbb{I}$ or $\mathbb{S}^1$ this has been made in [2]. As an example, for $k = 3$ and $\mathbb{X} = \mathbb{I}$ the frame of periods is

$$3 \cdot 3 \Rightarrow (3 \cdot 5 \Leftrightarrow 5) \Rightarrow (3 \cdot 7 \Leftrightarrow 7) \Rightarrow 3 \cdot 9 \Rightarrow (3 \cdot 11 \Leftrightarrow 11) \Rightarrow \cdots$$

$$3 \cdot 2 \cdot 3 \Rightarrow (3 \cdot 2 \cdot 5 \Leftrightarrow 2 \cdot 5) \Rightarrow (3 \cdot 2 \cdot 7 \Leftrightarrow 2 \cdot 7) \Rightarrow 3 \cdot 2 \cdot 9 \Rightarrow \cdots$$

$$\cdots$$

$$3 \cdot 2^k \cdot 3 \Rightarrow (3 \cdot 2^k \cdot 5 \Leftrightarrow 2^k \cdot 5) \Rightarrow (3 \cdot 2^k \cdot 7 \Leftrightarrow 2^k \cdot 7) \Rightarrow 3 \cdot 2^k \cdot 9 \Rightarrow \cdots$$

$$\cdots$$

$$\cdots \Rightarrow (3 \cdot 2^m \Leftrightarrow 2^m) \Rightarrow \cdots \Rightarrow (3 \cdot 2^2 \Leftrightarrow 2^2) \Rightarrow (3 \cdot 2 \Leftrightarrow 2) \Rightarrow 1.$$

We will deal with new situations where other frames can be obtained. It is the case when the state space is a continuum of dimension one (tree, graph, dendrite, etc). In particular, we will explicitly explore the case when $\mathbb{X}$ is a finite tree, since its periodic structure is known [1] and review what would be the situation in other cases.

[1] Ll. Alsedà, D. Juher and P. Mumbrú, *Periodic Behavior on Trees*, Ergod. Th. and Dynam. Sys., to appear.

[2] F. Balibrea and A. Linero, *On the Periodic Structure of Delayed Difference Equations of the Form* $x_n = f(x_{n-k})$ *on* $\mathbb{I}$ *and* $\mathbb{S}^1$, Journal of Difference Equations and Applications 9(3/4)(2003), 359-371.

François Blanchard

On the size of scrambled sets

coauthors: **W. Huang and Ľ. Snoha**

Consider a topological dynamical system (X, f). A subset S of X containing at least two points is called a *scrambled set* if for any $x, y \in S$ with $x \neq y$,

$$\liminf_{n\to\infty} d(f^n(x), f^n(y)) = 0 \tag{1}$$

and

$$\limsup_{n\to\infty} d(f^n(x), f^n(y)) > 0 \tag{2}$$

A system (X, F) is called *Li-Yorke chaotic* if it has an uncountable scrambled set.

Why uncountable? These notions were essentially developed in the context of interval maps, in which an uncountable scrambled set implies many other chaotic properties.

In this talk we shall develop several questions about scrambled sets in the general context of topological dynamics. There the assumption of Li-Yorke chaos, and also stronger ones like the existence of a Cantor scrambled set, or that of a residual scrambled set, or the fact that X itself is a scrambled set, are not so significant. But they provide valuable information in any case.

First, the following question arises naturally: is it true in general that a Li-Yorke chaotic system has a Cantor scrambled set, at least when the phase space is compact? This question is not answered completely but the answer is yes for symbolic systems, and when the system is weakly mixing or has positive entropy. There are severe restrictions on Li-Yorke chaotic dynamical systems without a Cantor scrambled set.

The second set of questions concerns the size of scrambled sets inside the space X itself. For which dynamical systems (X, f) do there exist first category, or second category, or residual scrambled sets, or a scrambled set which is equal to the whole space X?

It was previously known that interval maps cannot be residually scrambled, and that there exist completely scrambled systems (in which X is scrambled).

We prove that minimal compact systems, graph maps and a large class of symbolic systems containing subshifts of finite type are never residually scrambled. We also give examples of residually scrambled systems. some of them are strongly mixing and others have positive entropy; of systems with only first-catefgory scrambled sets; and, assuming the continuum hypothesis, of systems with second-category scrambled sets.

Jozef Bobok

On α-Lipschitz Extension of Transitive Interval Map with Topological Entropy $\log \alpha$

A (line) system is a pair $\langle T, g\rangle$, where $T \subset \mathbb{R}$ is compact and a map $g\colon T \to T$ is continuous. A system $\langle T, g\rangle$ is said to be transitive if for some $x \in T$ $\omega(g, x) = T$ - such a point is called transitive. Transitive systems $\langle T, g\rangle, \langle S, f\rangle$ are equivalent if there are transitive points $x_T \in T$ and $y_S \in S$ such that for every $i, j \in \mathbb{N}_0$

$$g^i(x_T) < g^j(x_T) \iff f^i(y_S) < f^j(y_S).$$

Obviously the topological entropies of equivalent transitive systems equal; assuming that $\langle T = [a, b], g\rangle$ is a transitive system with finite entropy $h(g) = \log \alpha$ we study when it is equivalent to a transitive system $\langle S, f\rangle$ with $f\colon\ S \to S$ α-Lipschitz.

Henk Bruin

Asymptotic arc-components of inverse limit spaces of unimodal maps

The inverse limit space of a unimodal map with a periodic turning point (of period n) is homeomprophic to the global attractor of certain Henon maps. This space has the local structure of a Cantor set cross an arc, except for n endpoints. But apart from these endpoints, there are other inhomogeneities in the form of asymptotic arc-components. Asymptotic arc-components were first discovered in these spaces by Barge & Diamond by the use of substitution tiling spaces. In this talk I want to present a different algorithm to find them and to describe their structure into fans and cycles. However, asymptotic arc-components can only be used for a partial classification of such inverse limit space, and hence gives no alternative approach to the classification question recently solved by Kailhofer.

Jean-Charles Delvenne

Decidability and Computational Universality in Symbolic Systems

We propose a definition of computational universality that applies to arbitrary discrete-time symbolic systems (i.e., continuous maps on a Cantor set). This definition generalizes universality of Turing machines and yields a new definition for universal cellular automata. It has the property to be robust with respect to noise on the initial condition., which is a desirable feature for physical realizability.

We derive necessary conditions for universality: for instance, a universal system has a sensitive point and a proper subsystem. We also exhibit a chaotic system that is universal, and discuss the hypothesis that the computation should only occur at the 'edge of chaos'.

We finally point out the difficulty to adapt this model of computation to analog systems.

This is a joint work with Prof. Petr Kurka and Prof. Vincent Blondel.

Tomasz Downarowicz

Expansiveness of certain Bratteli-Vershik diagrams

In a joint work with A. Maass we have proved that a minimal Cantor system represented by a Bratteli-Vershik diagram with bounded number of vertices is either an odometer or it is expansive (i.e., conjugate to a subshift). This sheds some light on the old problem of recognizing minimal subshifts by viewing their B-V diagram. During the talk I will try to present some details of the proof.

Fabien Durand

Cobham Theorems for dynamical systems and tilings

A. Cobham proved in 1969 a result about recognizability of sets of integers:

Let E be a set of positive integers. Let $p, q \geq 2$ be two multiplicatively independent integers. Then, the following statement are equivalent :

1) the set of the expansions in base p of the elements of E is recognizable by a finite automaton and the set of the expansions in base q of the elements of E is recognizable by a finite automaton ;

2) E is the union of a finite number of arithmetic progressions.

This result and its extensions have formulations in terms of (substitutive) dynamical systems. I will present a proof using symbolic dynamics or Bratteli diagrams.

Recently N. Ormes, C. Radin and L. Sadun on one part and C. Holton, C. Radin and L. Sadun proved two results on tilings that can be viewed as multidimensional Cobham Theorem. I will give some details on these theorems.

Juan Luis García Guirao

Three different types of chaos in discrete dynamical systems

The aim of this talk is to compare three different notions of chaos in discrete dynamical systems and analyzing the relations between them.

Eli Glasner

The local variational principle

The classical variational principle asserts that the topological entropy of a compact dynamical system $h_{\text{top}} = h_{\text{top}}(X,T)$, equals the supremum over the measure entropies h_μ where μ ranges over the set $M_T(X)$ of T invariant probability measures on X. The topological entropy h_{top} is defined as the supremum over the topological entropies $h_{\text{top}}(\mathcal{U})$ where $\mathcal{U}$ ranges over the finite open covers of X; however, given an open cover $\mathcal{U}$, the classical VP says little about the relation of $h_{\text{top}}(\mathcal{U})$ to measure theoretical entropy.

In a paper from 1997 Blanchard, Glasner and Host proved the existence of an invariant probability measure μ such that $h_\mu(\alpha) \geq h_{\text{top}}(\mathcal{U})$ for every finite Borel partition $\alpha \succ \mathcal{U}$.

In a recent work Glasner and Weiss show that

$$\sup_{\nu \in M_T(X)} \inf_{\alpha \succ \mathcal{U}} h_\nu(\alpha) \leq h_{\text{top}}(\mathcal{U}).$$

In particular for the BGH measure μ we then get a precise **local variational principle**: Given a finite open cover $\mathcal{U}$ of X there exists an invariant probability measure μ with $\inf_{\alpha \succ \mathcal{U}} h_\mu(\alpha) = h_{\text{top}}(\mathcal{U})$.

I will describe these and related results and discuss some applications.

Mark Holland

Rates of Mixing for one-dimensional Lorenz maps

We analyse the statistical properties of a class of one-dimensional Lorenz maps with critical points and singularities. Under suitable hypothesis we show that there is an ergodic measure, and the rate of mixing is exponentially fast. The techniques of proof involve the construction of an induced (countable) Markov map with exponential return time asymptotics.

Roman Hric

Unweighted dynamical zeta functions for piecewise monotone graph maps

In connection with the Entropy Conjecture it is known that the topological entropy of a continuous graph map is bounded from below by the logarithm of the spectral radius of the induced map on the first homology group. We show that in the case of a piecewise monotone graph map, its topological entropy is equal precisely to the maximum of the mentioned logarithm of the spectral radius and the exponential growth rate of the number of periodic points of negative type. This nontrivially extends a result of Milnor and Thurston on piecewise monotone interval maps. For this purpose we generalize the concept of Milnor-Thurston zeta function incorporating in the Lefschetz zeta function. We post some related results as well. This is a joint work with J. F. Alves and J. Sousa Ramos.

Victor Jimenez Lopez

Positive Lyapunov exponents or positive metric entropy imply sensitivity

Let $f : I = [0,1] \to I$ be a map and let μ be a probability measure on the Borel subsets of I absolutely continuous with respect to the Lebesgue measure and/or invariant for f. We say that $x \in I$ is *sensitive to initial conditions* if there exists a number $\delta > 0$ with the property that for any neighbourhood U of x there is some $n = n(x, U)$ such that $\mathrm{diam} f^n U > \delta$.

Under rather mild assumptions on f and μ we show that if the set of points with positive Lyapunov exponent has positive μ-measure or the metric entropy of f is positive then the set of points having sensitivity to initial conditions has positive μ-measure. In particular we improve previous results from [1].

This is an account of a joint work with Alejo Barrio Blaya, Universidad de Murcia, Spain [2].

[1] C. Abraham, G. Biau and B. Cadre, *On Lyapunov exponent and sensitivity*, J. Math. Anal. Appl. (2004) 395-404.

[2] A. Barrio Blaya and V. Jiménez López , *Positive Lyapunov exponents or positive metric entropy imply sensitivity* (2005), preprint.

Zdeněk Kočan

Triangular maps of the square

Let F be a triangular map $(x, y) \mapsto (f(x), g(x, y))$ from the unit square I^2 into itself. We consider a list of properties of this map, such as (i) F has zero topological entropy; (ii) period of any cycle of F is a power of 2; (iii) F has no homoclinic trajectory; (iv) $F|\ \mathrm{UR}(F)$ is non-chaotic in the sense of Li and Yorke; (v) $\mathrm{UR}(F) = \mathrm{Rec}(F)$; (vi) F is not DC1; (vii) F is not DC2; (viii) F is not DC3; and some others. Here DC1–DC3 denote distributional chaos of type 1–3. It is well-known that these properties are not mutually equivalent in this case (in contradistinction to the case $C(I, I)$). This talk is a survey of known relations between the properties in the case of triangular maps, and in the case of trigangular maps which are non-decreasing on the fibres.

Agnieszka Krause

Normalizers of Cantor minimal systems

The normalizer $N(T)$ and the full group $[T]$ of a homeomorphism T of a Cantor minimal set X are subgroups of the group $Pres_T(X)$ of all homeomorphisms of X preserving the invariant measures of T. Both those subgroups will be described for odometers

$$X = \{\sum_{t=0}^{\infty} x_t p_{t-1};\ p_{-1} = 1,\ p_t = \lambda_0 \cdots \lambda_t,\ \lambda_t \geq 2,\ 0 \leq x_t \leq \lambda_t - 1,\ t = 0, 1, \dots\}.$$

In particular if $\bar{\rho} = \{\rho_i^{(t)}\}$, $i = 0, 1, \dots, p_{t-1} - 1$, $t = 0, 1, \dots$ is a family of permutations $\rho_i^{(t)}\colon\ \{0, 1, \dots, \lambda_t - 1\} \to \{0, 1, \dots, \lambda_t - 1\}$ we define $S_{\bar{\rho}}(x) = \sum_{t=0}^{\infty} \rho_{\bar{x}_{t-1}}^{(t)}(x_t) p_{t-1}$, where $\bar{x}_{t-1} = \sum_{i=0}^{t-1} x_i p_{i-1}$, and then $S_{\bar{\rho}} \in Pres_T(X)$. Sufficient and necessary conditions for $S_{\bar{\rho}}$ to belong to $[T]$ or to $N(T)$ are given.

Jiří Kupka

The triangular maps with closed sets of periodic points

Let X, I and $C(X)$ denote a compact metric space, the unit interval $[0, 1]$ and the class of continuous maps of X into itself, respectively. A map $F \in C(X \times I)$ is called triangular if it is of the form $F(x, y) = (f(x), g_x(y))$, where $f \in C(X)$ and $g_x \in C(I)$ for every $x \in X$. Two apparently not related problems are studied for a given triangular map F. The first one is whether the existence of a regularly recurrent point of the base map f ensures the existence of some regularly recurrent point of the triangular map F. The second one was stated by A. N. Sharkovsky in the eighties and involves only the triangular maps on the square, i.e. $f \in C(I)$. For this base map f, there is a long list of properties equivalent to the condition that the periodic set of f is closed. We consider these properties for its simple extension, the triangular map F, and give an almost complete graph of relations among them.

Michal Kupsa

k-th limit laws of return and hitting times

We show that, for every k, the set of all possible k-limit laws of return times is equal to the set of all possible 1-limit laws, characterized by Lacroix (2002). We construct a rank one system where all these laws are realized along sequences of cylinders. In addition, we exhibit a link between k-limit laws of return and hitting times.

Petr Kůrka

Cellular automata in the space of Borel probability measures

We describe the statistical behaviour of a class of nonsurjective cellular automata using the concepts of particles and signals. The signals move through the cellular space with their particular speeds and transform to other signals when they meet one another. We describe the structure of the unique attractor of such a cellular automata in the space of the Borel probability measures.

Jan Kwiatkowski

Topologies on the groups of homeomorphisms and Borel automorphisms of a Cantor set

Let $Homeo(\Omega)$ be the group of all homeomorphisms of a Cantor set and let $Aut(X, \mathcal{B})$ be the group of all Borel automorphisms of a standard Borel space $(X, \mathcal{B})$. Several topologies on $Homeo(\Omega)$ and $Aut(X, \mathcal{B})$ and relations between them are considered. The closures of the most natural subsets of $Homeo(\Omega)$ and $Aut(X, \mathcal{B})$ are described. In particular, the closures of odometers, periodic, aperiodic, minimal, rank 1 homeomorphisms (the subsets of $Homeo(\Omega)$) and the closures of incompressible, smooth, odometers, periodic and aperiodic automorphisms (the subset of $Aut(X, \mathcal{B})$) are found.

Marek Lampart

Set-valued chaos

Let (X, d) be a compact metric space with a metric d and $f\colon\ X \to X$ a continuous map. Let us consider the space $(\mathcal{K}(X), d_H)$, where

$$\mathcal{K}(X) = \{K \subset X |\ K \text{ is compact}\}$$

and d_H is Hausdorff metric induced by d.

The main aim of this lecture is to discuss topological transitivity, weak mixing and mixing for (set-valued) map $\bar{f}\colon\ \mathcal{K}(X) \to \mathcal{K}(X)$ induced by f. Namely, it was proved by [A. Peris, Chaos, Solitons & Fractals - to appear] that the following three conditions are equivalent: (1) f is weakly mixing, (2) $\bar{f}$ is weakly mixing, (3) $\bar{f}$ is topological transitive, and also that f is mixing if and only if $\bar{f}$ is mixing. The assertion that $\bar{f}$ is topological transitive implies f is topological transitive was proved by [H. Román-Flores, Chaos, Solitons & Fractals 17 (2003) 99-104].

Finally, we prove that if f is chaotic in the sense of Li and Yorke then $\bar{f}$ is also chaotic in the sense of Li and Yorke.

Genadi Levin

Critical covering maps of the circle

I'll survey results about the dynamics of smooth covering maps of the circle that have a single (inflecting) critical point: rigidity, density of hyperbolicity, renormalizations and universality, existence of wild attractors etc.

Alejandro Maass

Continous eigenvalues of tiling systems

In this talk we give conditions under which a free minimal $\mathbb{Z}^d$-action on the Cantor set is a topological extension of the action of d rotations, either on the product $\mathbb{T}^d$ of d 1-tori or on a single 1-torus $\mathbb{T}^1$. We extend the notion of *linearly recurrent* systems defined for $\mathbb{Z}$-actions on the Cantor set to $\mathbb{Z}^d$-actions and we derive in this more general setting, a necessary and sufficient condition, which involves a natural combinatorial data associated with the action, allowing the existence of a rotation topological factor of one these two types.

Michal Málek

Omega limit sets and chaos on graphs

We give a full topological characterization of omega limit sets of continuous maps on graphs and we show that basic sets have similar properties as in the case of the compact interval. We also prove that the presence of distributional chaos, the existence of basic sets, and positive topological entropy (among other properties) are mutually equivalent for continuous graph maps.

Peter Maličký

Category version of the Poincare recurrence theorem

Ivo Marek

An aggregation variation on the Google matrix: Some of the mathematical aspects of the Google search engine

As well known, the Google search engine is modeled as a Markov chain whose states are single internet pages. After a brief description of the Google system some of its mathematical aspects will be discussed and namely the PageRank computing. The original approach of the creators of the Google system will be mentioned and some improved versions of various parts of the system presented. In particular, aggregation/disaggregation iterative methods will be shown as especially efficient tools for the PageRank computing. Finally, some interesting new results in Numerical Linear Algebra whose discovery has been motivated and initiated be the research around Google will be surveyed.

Michael Megrelishvili

Hereditarily non-sensitive dynamical systems

coauthor: **E. Glasner**

For an arbitrary topological group G any compact G-dynamical system (G, X) can be linearly G-represented as a weak*-compact subset of a dual Banach space V^*. As was shown in [3] the Banach space V can be chosen to be reflexive iff the metric system (G, X) is weakly almost periodic (WAP). We study [2] the wider class of compact G-systems which can be linearly represented as a weak*-compact subset of a dual Banach space with the Radon-Nikodým property. We call such a system a *Radon-Nikodým system* (RN). One of our main results is to show that for metrizable compact G-systems the three classes: RN, HNS (*hereditarily not sensitive*) and HAE (*hereditarily almost equicontinuous*) coincide. We investigate these classes and their relation to previously studied classes of G-systems such as WAP and LE (*locally equicontinuous* [1]). We show that the Glasner-Weiss examples of recurrent-transitive locally equicontinuous but not weakly almost periodic cascades are actually RN. Using fragmentability and Namioka's theorem we give an enveloping semigroup characterization of HNS systems and show that the enveloping semigroup of a compact metrizable HNS G-system is a separable Rosenthal compact, hence of cardinality $\leq 2^{\aleph_0}$. We investigate a dynamical version of the Bourgain-Fremlin-Talagrand dichotomy and a dynamical version of Todorčević dichotomy concerning Rosenthal compacts.

[1] E. Glasner and B. Weiss *Locally equicontinuous dynamical systems*, Colloq. Math. **84/85**, Part 2, (2000), 345-361.

[2] E. Glasner and M. Megrelishvili, *Linear representations of hereditarily non-sensitive dynamical systems*, submitted.

[3] M. Megrelishvili, *Fragmentability and representations of flows*, Topology Proceedings, **27:2** (2003), 497-544.

Mieczysław K. Mentzen

Cylinder cocycle extensions of minimal rotations, Part 2

coauthor: **A. Siemaszko**

Let X be a compact metric monothetic group and $T : X \longrightarrow X$ be a minimal rotation on X. Let $\varphi : X \longrightarrow \mathbb{R}$ be a continuous function. By a cylinder transformation we mean a homeomorphism $T_\varphi\colon X \times \mathbb{R} \longrightarrow X \times \mathbb{R}$ (or rather a $\mathbb{Z}$-action generated by it) given by the formula

$$T_\varphi(x, r) = (Tx, \varphi(x) + r).$$

The main result of the talk is the following.

Theorem 1. *Let T be a minimal rotation on the unit circle $\mathbb{T}$. If φ is a zero mean function of bounded variation and T_φ is topologically transitive then T_φ has no minimal subsets.*

By standard methods it is also shown that:

For every minimal rotation T on $\mathbb{T}$ there exists absolutely continuous zero mean function φ such that T_φ is topologically transitive.

A similar result as described in Theorem 1 holds when $\mathbb{T}$ is replaced by an odometer.

Using results described above it is shown the following.

Theorem 2. *Assume that X is infinite compact metric monothetic group. Let $T :\longrightarrow X$ be a minimal rotation. Then (X, T) admits a topologically transitive real cocycle.*

Some generalizations to the case of $\mathbb{R}^m$-cocycles in terms of *essential values* of cocycles are given.

Michał Misiurewicz

Rotation sets of billiards with one obstacle

We investigate the rotation sets of billiards on the m-dimensional torus with one small convex obstacle and in the square with one small convex obstacle. In the first case the displacement function, whose averages we consider, measures the change of the position of a point in the universal covering of the torus (that is, in the Euclidean space), in the second case it measures the rotation around the obstacle. A substantial part of the rotation set has usual strong properties of rotation sets.

Karl Petersen

Adic dynamics, random walks, and random permutations

In joint work with Sarah Bailey, Michael Keane, and Ibrahim Salama in several combinations, we have been studying a Bratteli-Vershik (adic) transformation based on a graph related to random permutations. What we call the Euler graph has vertices (n,k) for $n=0,1,2,\dots$ and $k=0,1,\dots,n$, with $k+1$ edges from (n,k) to $(n+1,k)$ and $n-k+1$ edges from (n,k) to $(n+1,k+1)$. Then paths from $(0,0)$ to (n,k) correspond to the permutations of $1,2,...,n+1$ with exactly $n-k$ falls and k rises. There is a natural symmetric measure on the space of paths defined by assigning equal weights $1/(n+2)$ to all of the edges downward from each each level n. By means of an argument involving random walks, we proved that this adic transformation is ergodic with respect to the symmetric measure. Recently we compared asymptotics of the Eulerian numbers found along different paths to prove the stronger result that in fact the symmetric measure is the unique fully supported ergodic invariant Borel probability measure for this system. This result has some of the flavor of a de Finetti-type theorem: if random permutations are being chosen in such a way that any two permutations of the same length which have the same number of rises are equally likely, and every permutation has positive probability, then in fact all permutations of the same length are equally likely. Many interesting dynamical, combinatorial, and probabilistic questions raised by this system and related ones remain open, and we continue to investigate them.

Feliks Przytycki

Entropy Conjecture

I will present results which I obtained jointly with Waclaw Marzantowicz. In 1974 Michael Shub asked the following question: When the topological entropy of a continuous mapping of a compact manifold into itself is estimated from below by the logarithm of the spectral radius of the linear mapping induced in the cohomologies with real coefficients? This estimate has been called Entropy Conjecture (EC). In 1977 I proved, jointly with Michał Misiurewicz, that EC holds for all continuous mappings of tori. In the talk I will sketch a proof that EC holds for all continuous mappings of compact nilmanifolds. I will sketch also another proof via Lefschetz and Nielsen numbers, under the assumption the map is not homotopic to a fixed points free map.

Peter Raith

Continuity of the topological pressure of a piecewise monotonic map under small perturbations.

Let $T:[0,1]\to[0,1]$ be a piecewise monotonic map, this means there exists a finite partition $\mathcal{Z}$ of $[0,1]$ into finitely many pairwise disjoint open intervals satisfying $\bigcup_{Z\in\mathcal{Z}} \overline{Z} = [0,1]$ such that $T|_Z$ is continuous and strictly monotonic for all $Z \in \mathcal{Z}$. An ε-perturbation of T is a piecewise monotonic map $\widetilde{T}$ having the same number of intervals of monotonicity as T, and the graph of $\widetilde{T}$ is contained in an ε-neighbourhood of the graph of T considered as a subset of $\mathbb{R}^2$.

Suppose that $f : [0,1] \to \mathbb{R}$ is continuous. Then the topological pressure is lower semi-continuous, if $p(T, f) > \sup f$. However, the topological pressure is not upper semi-continuous in general. In order to give upper bounds for the jumps up of the topological pressure a certain oriented graph $(\mathcal{G}, \to)$ is associated to T, and a matrix $G(f)$ is associated to f. Then

$$\limsup p(\widetilde{T}, \widetilde{f}) = \max\{p(T, f), \log r(G(f))\}.$$

Considering conditions implying the upper semi-continuity of the pressure for all continuous f leads to a surprising obstacle. There exist examples showing that $\log r\big(G(0)\big) = 0$ is not equivalent to the upper semi-continuity of the pressure for all continuous f. However, for "reasonable" continuous functions it is equivalent. In the general case a condition equivalent to the upper semi-continuity is that every cycle in $(\mathcal{G}, \to)$ is a "periodic orbit" of T. For continuous T the upper semi-continuity is equivalent to the condition that no endpoint (except 0 and 1) of an interval of monotonicity of T is periodic.

Michał Rams

Contracting on average baker maps

The random iterated function system is a finite family of maps $\{f_i\} : \mathbb{R} \to \mathbb{R}$ that we can choose with a given probabilities $\{p_i\}$. The system is called contracting on average if $\sum p_i \log \mathrm{Lip}(f_i) < 0$. I am going to present estimations on the Hausdorff dimension of the (unique) invariant measure of the system: from above (improving previous results of Nicol, Sidorov, Broomhead and Fan, Simon, Toth) and below (under some geometrical separation condition on the system).

Ana Rodrigues

Secondary Bifurcations in Systems with $\mathbf{S}_N$-Symmetry

In their recent paper, Dias and Stewart (Secondary Bifurcations in Systems with All-to-All Coupling, *Proc. R. Soc. Lond. A* (2003) **459**, 1969-1986.) studied the existence, branching geometry, and stability of secondary branches of equilibria in all-to-all coupled systems of differential equations, that is, equations that are equivariant under the permutation action of the symmetric group $\mathbf{S}_N$. They consider the general cubic order truncation system of this type. Primary branches in such systems correspond to partitions of N into two parts p,

q with $p + q = N$. Secondary branches correspond to partitions of N into three parts a, b, c with $a + b + c = N$. They prove that when all of the a, b, c are different from $N/3$ secondary branches exist, and are (generically) globally unstable in the cubic-order system. In this work they realized that the cubic order system is too degenerate to provide secondary branches if $a = b = c$. In this paper we prove the existence and the branching geometry of secondary branches of equilibria with $\mathbf{S}_n \times \mathbf{S}_n \times \mathbf{S}_n$ symmetry, in systems of ordinary differential equations that commute with the permutation action of the symmetric group $\mathbf{S}_{3n}$ (action on $\mathbf{R}^{3n}$). Moreover, we prove that the solutions of the secondary branch are unstable.

Pierre-Paul Romagnoli

Conditional Entropy Pairs and Applications

We introduce several notions of pairs including topological conditional entropy pairs as a generalization of topological entropy pairs defined in [3] extending the notions introduced in [2] and we use this to characterize the factors of a topological dynamical system according to their topological entropy. We also give a measure theoretical version and obtain conditional variational principle also proving equality of the "+" and "−" notions given in [1].

[1] P.-P. Romagnoli, *A local variational principle for the topological entropy,* Ergodic Th. and Dynam. Sys. 23 (2003), 1601-1610.

[2] M. Misiurewicz, *Topological conditional entropy*, Studia Math. 55 (1976), 176-200.

[3] F. Blanchard, *Fully positive topological entropy and topological mixing,* Contemporary Mathematics 135 (1992), 95-105.

Michael Schraudner

On the canonical-boundary representation for automorphism groups of locally compact countable state Markov shifts

Using the canonical compactification defined by D. Fiebig and U.-R. Fiebig we introduce the canonical-boundary representation for automorphism groups of locally compact countable state Markov shifts and study its range. This homomorphism is a conjugacy invariant capturing information about the symmetry of the Markov shift near its (canonical) boundary. It exhibits which actions on the boundary can be realized by automorphisms.

In a second step we give another new conjugacy invariant: The "path-structure at infinity", which is a certain relation on the orbits of the canonical boundary originating from the structure of some graph presentation near each of these orbits. This invariant is stronger than the canonical boundary and the periodic data at infinity. If time permits we will determine its influence on the range of the canonical-boundary representation and on the extendability of automorphisms from subsystems to the entire locally compact countable state Markov shift.

Artur Siemaszko

Cylinder cocycle extensions of minimal rotations, Part 1

coauthor: **M.K. Mentzen**

Let X be a compact metric monothetic group and $T : X \longrightarrow X$ be a minimal rotation on X. Let $\varphi : X \longrightarrow \mathbb{R}$ be a continuous function. By a cylinder transformation we mean a homeomorphism $T_\varphi \colon X \times \mathbb{R} \longrightarrow X \times \mathbb{R}$ (or rather a $\mathbb{Z}$-action generated by it) given by the formula

$$T_\varphi(x, r) = (Tx, \varphi(x) + r).$$

The main result of the talk is the following.

Theorem 1. *Let T be a minimal rotation on the unit circle $\mathbb{T}$. If φ is a zero mean function of bounded variation and T_φ is topologically transitive then T_φ has no minimal subsets.*

By standard methods it is also shown that:

For every minimal rotation T on $\mathbb{T}$ there exists absolutely continuous zero mean function φ such that T_φ is topologically transitive.

A similar result as described in Theorem 1 holds when $\mathbb{T}$ is replaced by an odometer.

Using results described above it is shown the following.

Theorem 2. *Assume that X is infinite compact metric monothetic group. Let $T :\longrightarrow X$ be a minimal rotation. Then (X, T) admits a topologically transitive real cocycle.*

Some generalizations to the case of $\mathbb{R}^m$-cocycles in terms of *essential values* of cocycles are given.

Károly Simon

The algebric difference of random Cantor set

We consider some families of random Cantor sets on the line and investigate the question whether the condition that the sum of the Hausdorff dimension is larger than one, implies the existence of interior points in the difference set.

Jaroslav Smítal

Dynamics of triangular maps - recent progress

Ľubomír Snoha

Noninvertible minimal maps revisited

The talk is based on several papers which were published or submitted during the last 5 years.

It is discussed how much non-injective a minimal map can be from the topological point of view and from the point of view of invariant measures. Some examples and methods of construction of noninvertible minimal maps will also be recalled.

Vladimír Špitalský

Omega-limit sets in dendrites

For any fixed dendrite X we give the topological characterization of the system of all ω-limit sets of all continuous selfmaps of X. Namely we show that a nonempty closed subset M of X is an ω-limit set of some continuous selfmap $f : X \to X$ if and only if either it is a finite union of nondegenerate subdendrites or it is a nowhere dense subset of X which does not satisfy the following condition: M has countably many components, only finitely many of them are nondegenerate and M has an isolated nondegenerate component.

Jerzy Szymanski

Extreme relations for topological flows

The extreme relation (extreme with respect to an invariant measure μ) for a topological flow (X, T) is a relation $R \in CER(X)$ satisfying conditions (i) $(T \times T)(R) \subset R$, (ii) $\bigcap_{n=0}^{\infty} (T \times T)^n(R) = \Delta$, (iii) $\bigvee_{n=0}^{\infty} (T \times T)^{-n}(R) = \Pi(T)$ $(\Pi_\mu(T))$, where the symbol $\Pi(T)$ $(\Pi_\mu(T))$ stands for Pinsker relation (Pinsker relation with respect to μ). The existence of such a relation in every topological flow for any invariant ergodic measure is shown. This result can be regarded as the topological analogue of Rokhlin-Sinai theorem for measurable partitions. It follows, among other things, that any topologically deterministic flow has zero topological entropy and any measure-theoretic K-system (X, T) w.r. to an ergodic measure with full support is a topological K-flow.

Klaus Thomsen

On the ergodic theory of synchronized systems

A synchronized system is a twosided shift space which admits a synchronizing word. They form a class of shift spaces which generalize sofic shifts, and have many features in common these. In particular, they can be presented by a (generally infinite) labeled graph, called the Fischer cover of the shift space. I will describe in which way, and to which extend, the Choquet simplex of shift invariant Borel probability measures of a synchronized system can be described through the Fischer cover. It turns out that the structure of irreducible components which show up also for shift spaces that are irreducible, but not of finite type, provide a key to the structure of this simplex. An emphasis will be put on the existence of ergodic measures of full support and measures of maximal entropy.

Michael Todd

Good invariant measures on the Julia sets of polynomials

Suppose that a polynomial has a dendrite Julia set and positive Lyapunov exponents. Starting with conformal measure, we use a tower construction to find an invariant measure. This measure has nice properties: for example almost every point goes to large scale with positive frequency.

Marcin Wata

Dimension and infinitesimal groups of topological dynamical systems

The dimension group $K^0(X,T)$ of a Cantor minimal system (X,T) is the quotient group $C(X,\mathbb{Z})/B_T$, where $B_T = \{f - f \circ T^{-1},\ f \in C(X,\mathbb{Z})\}$. The infinitesimal group $Inf(X,T)$ of (X,T) is the quotient group $N(X,T)/B_T$, where

$$N(X,T) = \{f \in C(X,\mathbb{Z}) : \int_X f d\mu = 0 \text{ for every } \mu \in M(X,T)\}.$$

The dimension groups and the infinitesimal groups of Cantor minimal systems play important role in the orbital theory in topological dynamics. Methods of selecting sequences of homomorphisms determining the dimension and infinitesimal groups of (X,T) based on non-proper Bratteli diagrams are described.