Problems and recent results in single recurrence

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The Poincaré Recurrence Theorem says that whenever (X, μ, T) is a probability measure preserving system and $A \subset X$ has positive measure, there is an $n \in \mathbb{N}$ such that $\mu(A \cap T^{-n}A) > 0$. H. Furstenberg and A. Sárközy independently proved an appealing refinement: under the same hypotheses, one can conclude that $\mu(A \cap T^{-n^2}A) > 0$ for some $n \in \mathbb{N}$.

One may ask, in general, for which sets $S \subset \mathbb{N}$ can one conclude that $\mu(A \cap T^{-n}A) > 0$ for some $n \in S$? We call such an S a set of measurable recurrence. Many examples and non-examples are known, but a satisfying description remains elusive.

We will survey some of the classical examples of sets of recurrence, motivating several open problems. We will also summarize a technique for building Rohlin towers with special properties, leading to interesting non-examples of sets of recurrence.