$\bar{d}$-approachability, entropy density and $\mathscr{B}$-free shifts

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Let $\bar{d}$ denote the pseudometric on the full shift over a finite alphabet A given by the upper asymptotic density of the set of positions at which two A-valued sequences differ. Write $\bar{d}^H$ for the associated Hausdorff pseudometric for subsets of the full shift. We study which properties of shift spaces (shifts) are closed with respect to $\bar{d}^H$. In particular, we study shifts, which are $\bar{d}^H$ limits of their Markov approximations. We call these shifts $\bar{d}$-approachable. We provide a topological characterization of chain mixing $\bar{d}$-approachable shifts analogous to Friedman-Ornstein's characterization of Bernoulli processes. We prove that many specification properties imply $\bar{d}$-approachability. It follows that mixing shifts of finite type, mixing sofic shifts, and beta-shifts are $\bar{d}$-approachable. We construct minimal and proximal examples of mixing $\bar{d}$-approachable shifts. We also show that $\bar{d}$-approachability and chain-mixing imply $\bar{d}$-stability, a property recently introduced by Tim Austin. This leads to examples of minimal or proximal $\bar{d}$-stable shift spaces, answering a question posed by Austin. Finally, we show that the set of shifts with entropy-dense ergodic measures is $\bar{d}^H$ closed. Note that entropy-density of ergodic measures is known to follow from the specification property, but the minimal or proximal examples are far from having any specification. Finally, we show entropy-density for a class of shifts that includes many interesting $\mathscr{B}$-free shifts. These shift spaces are not $\bar{d}$-approachable, but they are $\bar{d}^H$ limits of sequences of transitive sofic shifts, and this implies entropy-density.

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