On self-similar continua with finite intersection property

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Let $\mathcal{S} = \{S_1, ..., S_m\}$ be a system of injective contraction maps in $\mathbb{R}^n$, whose attractor K is connected. If for any non-equal $i, j \in \{1, ..., m\}$, the intersection of the pieces K_i and K_j of the attractor K contains at most s points, then we say that the system $\mathcal{S}$ is a FI(s)-system of contractions and we say that K is a self-similar continuum with finite intersection property.

For a FI(s)-system of contractions $\mathcal{S}$ we define its bipartite intersection graph $\Gamma(\mathcal{S})$. This graph determines the topological properties of the attractor K. Particularly, we prove the following Theorem.

Theorem 1. *Let $\mathcal{S}$ be a system of injective contraction maps in a complete metric space X, which satisfies finite intersection property. The attractor K of the system $\mathcal{S}$ is a dendrite iff the intersection graph $\Gamma(\mathcal{S})$ of the system $\mathcal{S}$ is a tree.*

We prove that if $\mathcal{S}$ is a FI(s)-system of similarities in $\mathbb{R}^n$ which satisfies open set condition, then there is a finite upper bound for the numbers of addresses $\#\pi^{-1}(x)$ and $\#\pi^{-1}(\partial K_{\mathbf{j}})$; for the ramification numbers $N_C(\{x\})$ and $N_C(K_{\mathbf{j}})$ and for the topological order $\mathrm{Ord}(x, K)$ and $\mathrm{Ord}(K_{\mathbf{j}}, K)$.

Acknowledgements: This work was supported by the Mathematical Center in Akademgorodok with the Ministry of Science and Higher Education of the Russian Federation, agreement no. 075-15-2019-1613.