Levels of local chaos for special Blocks Families and applications for Turing Machine

Levels of sensitive for Turing Machine

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Outline

Questions:

(1) Is it always true that a sensitive subsystem leads to a general sensitive system?

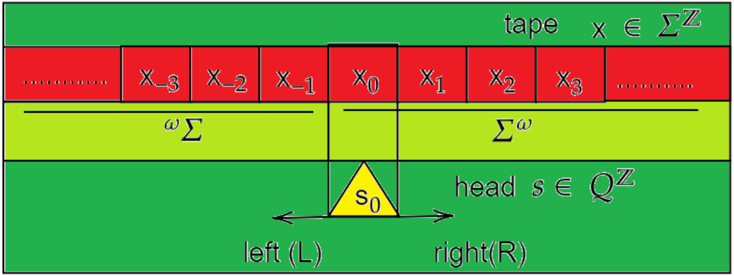
(2) Can a certain level of chaoticity be determined by particular conditions of sensitivity?

Objective:

- Study the sensitivity levels of Turing Machines in TDS.

Generalizations About Topological Dynamical Systems in Turing Machines

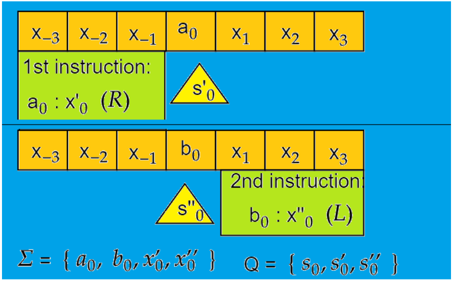
definition



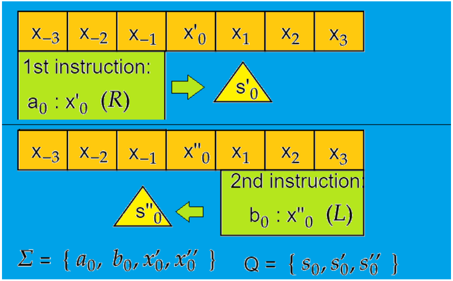
- The Turing machine is an abstract mathematical computational model consisting of a tape of infinite length, divided into cells with a given input and a head that reads the input tape, which was introduced in 1936.

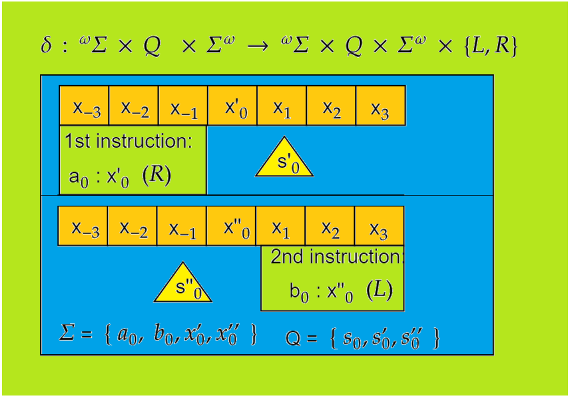
- The study of the Turing machine from the point of view of dynamics systems was initiated by Moore [39] and first studied by Kurka [32, 33].
- Separately, both authors formalized three dynamic models: *Generalized Shifts*, *Turing Machine with Moving Head* and *Turing machine with moving tape*, all abbreviated as *GS*, *TMH* and *TMT*.

Classical Movement

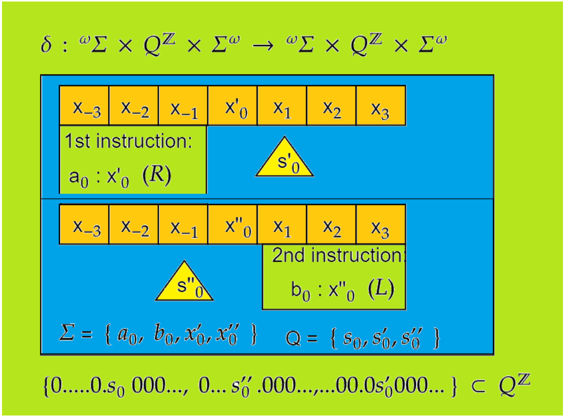


- After reading an input symbol, it is replaced with another symbol, its internal state is changed, and it is moved from one cell to some direction (*left (L)* or *right (R)*).


x_{-3} x_{-2} x_{-1} x'_0 x_1 x_2 x_3
1st instruction:
$a_0 : x'_0 \ (R)$
s'_0
x_{-3} x_{-2} x_{-1} x''_0 x_1 x_2 x_3
2nd instruction:
$b_0 : x''_0 \ (L)$
s''_0
$\Sigma = \{ a_0, b_0, x'_0, x''_0 \}$
$Q = \{ s_0, s'_0, s''_0 \}$

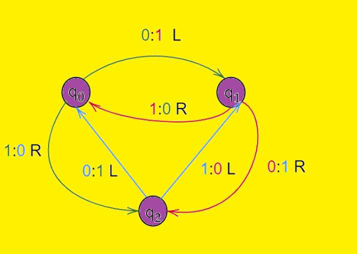


- Formally, the classical Turing machine is composed of a finite set of states (denoted by Q),on alphabetic tape (denoted by Σ), an alphabetic entry, a transition map ($\delta : Q \times \Sigma \to Q \times \Sigma \times \{-1, 0, 1\}$, where the set with -1,0 and 1 represents the directions), an initial and final state and a blank symbol



- Here the instruction is modified, replacing the direction of the instruction, by all the configurations of Q. Furthermore, here there is no a blank symbols, just two symbols $\Sigma = \{0, 1\}$ and also, it doesn't stays in the same position by movement.

Example of Turing Machine



- An example of how the machine works is a machine with a finite set Q with three states (denoted as $Q = \{q_0, q_1, q_2\}$, with the initial state q_i and the final state q_f), a set Σ with two symbols or letters (also written as $\Sigma = \{0, 1\}$), with input and output alphabets x_i and x_f. Then, the instruction is to change x_i to x_f, moving the head, into the right or left, changing the state q_i to q_f. In our scheme, the formal instructions seems as:

$\delta(..x_{-2}x_{-1}, \{s_i\}_{i\in\mathbb{Z}_-}, q_0, \{s_i\}_{i\in\mathbb{N}}, .0x_1x_2..) =$ $(..x_{-2}x_{-1}, \{s_{i-1}\}_{i\in\mathbb{Z}_-}, q_1, \{s_{i-1}\}_{i\in\mathbb{N}}, .1x_1x_2..)$
$\delta(..x_{-2}x_{-1}, \{s_i\}_{i\in\mathbb{Z}_-}, q_0, \{s_i\}_{i\in\mathbb{N}}, .1x_1x_2..) =$ $(..x_{-2}x_{-1}, \{s_{i+1}\}_{i\in\mathbb{Z}_-}, q_2 \{s_{i+1}\}_{i\in\mathbb{N}}, .0x_1x_2..)$
$\delta(..x_{-2}x_{-1}, \{s_i\}_{i\in\mathbb{Z}_-}, q_1, \{s_i\}_{i\in\mathbb{N}}, .0x_1x_2..) =$ $(..x_{-2}x_{-1}, \{s_{i+1}\}_{i\in\mathbb{Z}_-}, q_2, \{s_{i+1}\}_{i\in\mathbb{N}}, .1x_1x_2..)$
$\delta(..x_{-2}x_{-1}, \{s_i\}_{i\in\mathbb{Z}_-}, q_1, \{s_i\}_{i\in\mathbb{N}}, .1x_1x_2..) =$ $(..x_{-2}x_{-1}, \{s_{i+1}\}_{i\in\mathbb{Z}_-}, q_0, \{s_{i+1}\}_{i\in\mathbb{N}}, .0x_1x_2..)$
$\delta(..x_{-2}x_{-1}, \{s_i\}_{i\in\mathbb{Z}_-}, q_2, \{s_i\}_{i\in\mathbb{N}}, .0x_1x_2..) =$ $(..x_{-2}x_{-1}, \{s_{i-1}\}_{i\in\mathbb{Z}_-}, q_0, \{s_{i-1}\}_{i\in\mathbb{N}}, .1x_1x_2..)$
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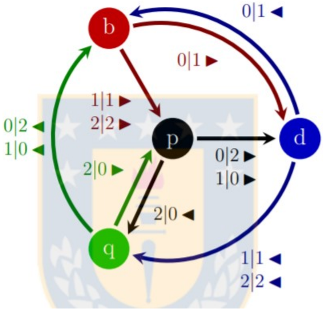
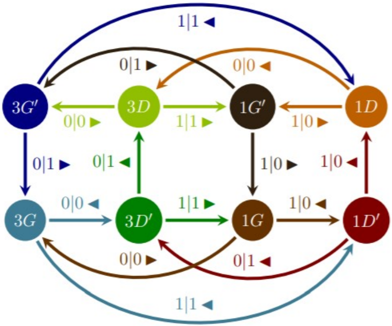
Remark 1: To obtain a better observation of the classic Machine, replace all the elements, which do not point to the current symbol by zero, and then project on the head settings.





Other examples are *SMART* and *BINSMART* machines. The fist one contains four states, three symbols $(0, 1, 2)$ and representing as a dynamical system, is *aperiodic*, *symmetric* and *minimal* (then *transitive*). The second one, contains eight states, two symbols $(0, 1)$ and it's *transitive*. For more details, read the results of [4, 5].

- En [30, 32] The properties of sensitive and chaos are studied in *Turing machines* and *Cellular Automata* among some.

Sensitive in Turing Machine

In [30] the sensitivity system according to Turing Machines was defined as:

The system (X, F) is *sensitive* if there exists a finite observation window

$W \subset \mathbb{Z}$, such that:

$\forall E \subseteq \mathbb{Z} : \forall x \in X, \exists y \in X, \exists n \in \mathbb{N} : x_E = y_E$ and $(F^n(x))_W \neq (F^n(y))_W$.

We'll call to this system *classical sensitive wrt TM*

Next, let's defines a shift map $\sigma : \Sigma^{\mathbb{N}_0} \to \Sigma^{\mathbb{N}_0}$, where $\sigma(x)_i = x_{i+1}$. About the metric, which it denotes $d_{\Sigma^{\mathbb{N}_0}}$, it can be represented as:

$$d_{\Sigma^{\mathbb{N}_0}}(\sigma^n(x), \sigma^n(y)) = \begin{cases} 0 & , x_{i+n} = y_{i+n}, i \geq 0 \\ 2^{-\inf\{j \geq 0 : x_{j+n} \neq y_{j+n}\}} & , x_{i+n} \neq y_{i+n}, i \geq 0 \end{cases} \tag{1}$$

Clearly, by talking about sensitive, there is a constant $M \in \mathbb{N}_0$, such that for $n \in \mathbb{N}$, $M > \inf\{j \geq 0 : x_{j+n} \neq y_{j+n}\}$. For this case, We are going to refer the system $(\Sigma^{\mathbb{N}_0}, \sigma)$ as *σ-sensitive*

With respect of the Turing Machine's dynamical system, the metric on $\mathcal{M}_\omega =^\omega \Sigma \times Q^{\mathbb{Z}} \times \Sigma^\omega$, which it denotes as $d_{\mathcal{M}_\omega}$, has the form:

$$d_{\mathcal{M}_\omega}(\delta^n(a), \delta^n(n)) = \begin{cases} 0 & , a_{i+n} = b_{i+n}, i \geq 0 \\ 2^{-\inf\{j \geq 0 : a_{j+n} \neq a_{j+n}\}} & , b_{i+n} \neq b_{i+n}, i \geq 0 \end{cases}, \tag{2}$$

For $a, b \in \mathcal{M}_\omega$. Similarly, by talking about *sensitive*, there is a constant $M' \in \mathbb{N}_0$, such that for $n \in \mathbb{N}$, $M' > \inf\{j \geq 0 : a_{j+n} \neq b_{j+n}\}$. For this case, We are going to refer the system $(\mathcal{M}_\omega, \delta)$ as δ-*sensitive*.

Remark 2: $\delta := \delta_{\omega_\Sigma} \times \delta_{Q^{\mathbb{Z}}} \times \sigma \in C(\mathcal{M}_\omega)$, where
$\delta_{\omega_\Sigma} :^\omega \Sigma \rightarrow^\omega \Sigma$,
$\delta_{Q^{\mathbb{Z}}} : Q^{\mathbb{Z}} \rightarrow Q^{\mathbb{Z}}$
$\delta_{\Sigma^\omega} : \Sigma^\omega \rightarrow \Sigma^\omega$,
About the system with the maps δ_{ω_Σ} and δ_{Σ^ω} are quite similar to σ, since the elements are in Σ, but with this map. Otherwise, the metric over $Q^{\mathbb{Z}}$ is just *discrete*, since the states of two systems could be the same or not.

Result 1

Proposition 1

Let $(\mathcal{M}_\omega, \delta)$ be a system. Then, the following statements are true:

(1) *The system is δ-sensitive*

(2) *The system has at least one map conforming δ, which is sensitive.*

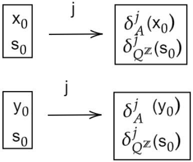
Idea of the proof: (2) $\Rightarrow$ (1) is obvious. Conversely, choose some of the maps in any case. the head depends of the elements of Σ (in particular, of the element, which is above of it). However, the sensitive properties are not directly related, since the movement is because the action generated by δ_{Σ^ω} and δ_{ω_Σ}. Then, it's easy to check that:

- δ_{Σ^ω}-sensitive $\Rightarrow$ $\delta_{Q^\mathbb{Z}}$-sensitive
- δ_{ω_Σ}-sensitive $\Rightarrow$ $\delta_{Q^\mathbb{Z}}$-sensitive

However, conversely it's not true.

Result 1

Now, Fixing $x_0 \in \Sigma$ and $i \in \mathbb{Z}$, we have a state s_0 and an instruction of the Machine, which always connect x_i with s_i. The same can be applying for y_i and s_i. Then, fixing $j \in \mathbb{Z}$, as the first element, where $\delta_{Q^{\mathbb{Z}}}(...0.s_0...) \neq \delta_{Q^{\mathbb{Z}}}(...0.s_0...)$, it doesn't implies that the elements of the tape are distinct, but it implies that the elements of $\mathcal{M}_\omega$ are separated. Since the tape doesn't change, then j is also the first element also for δ. Using the definitions of below, we can complete the proof.



Define j = M-2, where M $\geqslant 1$

Choose n $\geqslant j$

About Question 1

Now, suppose that $\delta^{l_i}_{\Sigma^\omega} := \sigma^i$, for $l : \mathbb{N} \to \mathbb{N}$, with $l_i < l_{i+1}$ for any $i \in \mathbb{N}$. Then, since we have M as the constant of sensitive for $(\Sigma^{\mathbb{N}_0}, \sigma)$, then we have some $n \in \mathbb{N}$, such that $l(M) > l_{i+n}$. Since the system is δ-sensitive, we fix M', where $M' > l_{i+n}$. However, clearly, in the whole process, we start assuming that if the machine perfectly imitates to the shift map, then $l_i = i$, but in general $l_i \geq i$ for any $i \in \mathbb{N}$. Having this idea, is easy to affirm that if system is δ-sensitive, then is σ-sensitive. Otherwise, it's not so clear to prove the converse part, but we talk about it below.

Furstenberg family

Let $\mathbb{N}$ the set of positive integers and $\mathcal{P}$ the collection of all subsets of $\mathbb{N}$. A collection $\mathcal{F} \subset \mathcal{P}$ is a Furstenberg family if it's hereditary upwards (i.e $F_1 \subset F_2,\ F_1 \in \mathcal{F} \Rightarrow F_2 \in \mathcal{F}$)

- The dual family is $\mathcal{F}^* = \{A \subset \mathbb{N} : \mathbb{N} \setminus A \notin \mathcal{F}\}$
- $\mathcal{F}$ is proper if it is a nonempty proper set of the family P (i.e $F \in P$, but $\emptyset \notin \mathcal{F}$)
- Set $\mathcal{F}_{inf}$ the collection of all infinite subsets of $\mathbb{N}$
- $\mathcal{F}_t = \{A \subseteq \mathbb{N} : (\forall m \in \mathbb{N})(\exists m \in \mathbb{N}) : \{m, m+1, .., m+n\} \subset A\}$ *thick*
- $\mathcal{F}_s$ the collection of all syndetic subsets of $\mathbb{N}$, i.e $\mathcal{F}_s = \mathcal{F}_t^*$
- $\mathcal{F}_{ps}$ the collection of all piece wise syndetic subsets of $\mathbb{N}$, i.e $\mathcal{F}_{ps} = \{F \cap G \in \mathcal{F} : F \in \mathcal{F}_t \wedge G \in \mathcal{F}_s\}$

Block family

Classical Definition and some results [5]

Let $\mathcal{F}$ be a family. The *block family* of $\mathcal{F}$, denoted by $b\mathcal{F}$ can be written as the collection:

$$b\mathcal{F} = \{A \subset \mathbb{N} : \exists F \in \mathcal{F}, (\forall m \in \mathbb{N})(\exists a_m \in \mathbb{N}) : a_m + [0, m] \cap F \subset A\}$$

Some results about the block family are in [5]

(1) $b(b\mathcal{F}) = b\mathcal{F}$

(2) $b\mathcal{F}_{inf} = \mathcal{F}_{inf}$

(3) $b\mathcal{F}_s = b\mathcal{F}_{ps} = b\mathcal{F}_{pubd}$

(4) $b\mathcal{F}^*_{inf} = \mathcal{F}_t$

Remark 1: Since (2) is true, now suppose that $\mathcal{A} \subset \mathcal{F}_{inf}$, that doesn't imply neither $b\mathcal{A} = \mathcal{A}$, nor $\mathcal{A}$ is really a family. Anyway, we denote $\mathcal{A}$ as a *sub-collection of family* of $\mathcal{F}_{inf}$.

Definition of $\mathcal{F} - Sensitive$

Definition 2

Let $({}^{\omega}\Sigma \times Q^{\mathbb{Z}} \times \Sigma^{\omega}, \delta)$ be a system. The system is $\mathcal{F} -$ *sensitive* if for a finite observation window $W \subset \mathbb{Z}$, the set $N(E, W)$, which is equals to

$$\left\{ n \in \mathbb{N} : E \subseteq \mathbb{Z} : x, y \in^{\omega} \Sigma \times Q^{\mathbb{Z}} \times \Sigma^{\omega} : \; x_E = y_E, (F^n(x))_W \neq (F^n(y))_W \right\}$$

belongs to $\mathcal{F}$

About the Machine

Let $\{s_i\}_{i\in\mathbb{Z}} \in Q^{\mathbb{Z}}$, with a finite number of non null states (i.e $s_j \neq 0$ for $j \in A$ with $A \subset \mathbb{Z}$ as finite). Then, the Turing Machine can be represented as:

$$\begin{pmatrix} \ldots x_{-2}x_{-1}x_0x_1x_2x_3\ldots x_j\ldots \\ 00\ldots\ldots s_0\ldots\ldots\ldots 0..0 \end{pmatrix} \in \mathcal{M}_\omega \tag{3}$$

where only the central position 0 has a state at a given instant, and hence the head's tape below is a finite cylinder, which goes changing while the tape is shifting under the action of the transition map.

- Next, when transition map is applied over $\mathbb{N}$, the state starts to change, however, although the state changes, there is only one state in the configuration below.
- Furthermore, depending on the initial state of the symbols, the instruction leads the head to move in one direction, while the symbol changes to a known value.

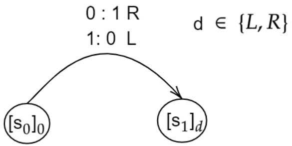
Before to continue, let define $[a_1 ... a_M]_{r,\Sigma}$ and $[b_1 .... b_{M'}]_{r',Q}$ as cylinders over $\Sigma^{\mathbb{Z}}$ and $Q^{\mathbb{Z}}$, in where:

$$[a_1 ... a_M]_{r,\Sigma} = \left\{ x \in \Sigma^{\mathbb{Z}} : x_{i+r} = a_i, i \in [[1, M]] \right\} \tag{4}$$

$$[b_1 ... b_{M'}]_{r',Q} = \left\{ s \in Q^{\mathbb{Z}} : s_{i+r'} = b_i, i \in [[1, M']] \right\} \tag{5}$$

An example, if $x_0 = 0$ and $\{s_0, s_1\} \subset Q$, suppose that the instruction for move the tape in join with the head from $[s_0]_{0,Q}$ to $[s_1]_{1,Q}$ is $0 : 1$ to the right. Otherwise, if $x_0 = 1$ and it's above s_0, the instruction is $1 : 0$ to the left.

Then, the sub graph of the machine can be expressed as:



$$\delta([0]_{0,\Sigma}, [s_0]_{0,Q}) := ([1]_{0,\Sigma}, [s_1]_{1,Q})$$
$$\delta([1]_{0,\Sigma}, [s_0]_{0,Q}) := ([0]_{0,\Sigma}, [s_1]_{-1,Q})$$

- For this occasion, the transition map was modified to include the direction in terms of finite cylinders. This process preserves the classical results if there are a finite number of states.

Now, I'm going to assume that $x \in [a_0a_1...a_j]_0$ for $j \in \mathbb{N}$, then there is a time of process, who depends of the position of the initial tape and the internal process of the machine. Fixing a position $i \in \mathbb{N}$, the number of step of time of process, denoted as $l_i : \mathbb{Z} \to \mathbb{N}$, can be expressed as:

$$\begin{pmatrix} a_i & a_{i+1}...a_j \\ s_{l_i} & 0......... \end{pmatrix} \dashv^{l_{i+1}} \begin{pmatrix} a_{i+1} & a_{i+2}...a_j \\ s_{l_{i+1}} & 0......... \end{pmatrix} \tag{6}$$

where, for $i + 2 < j$ and $i \in \mathbb{N}$.This type of Turing Machine, I'll call it *system with rule 1*, which the rules of the rule 1 are (4) and the properties of our new δ.

- Naturally, the sequence $(l_i)_{i\in\mathbb{Z}}$ is increasing over $\mathbb{N}$.
- Clearly, in this process with have a dynamic generated by a sub-orbit of $(\Sigma^{\mathbb{N}}, \sigma)$, which it preserves the properties of our interesting, since my intention is to study forward orbits.

Result about sensitive system for one sided full shift

- Suppose that there is two Turing machines equals between them on the first j terms. When $j = inf\,\{m \geq 0 : x_l \neq y_l\}$ (or in the classical sense, I define W with $\{j\} \subset W$.
- For the classical case, it is enough to know that W has only one element, to prove that the system is *sensitive*. However and more interesting, we can find out the number of steps, just with find the existence of instructions where the above moves the tape through a series of processes. Hence, we must to compute the length of the cylinder in which the rules lead to the observed sub process.

Result 2

Proposition 3

Let $(\mathcal{M}_\omega)^{x2}, \delta^{x2})$ be the product of the system, with each one having the rule 1. Then there exists a sub sequence $\{l_i\}_{i=0}^{\infty}$, such that there exists at least one element of $FS(\{l_i\}_{i=0}^{\infty})$, which is into the set $N(W, E)$

I prove this proposition, using the following corollary

Corollary 4

Let $[\bar{x}^{j-1}a_j]_{0,\Sigma} \times [\bar{x}^{j-1}b_j]_{0,\Sigma} \subset \Sigma^{\mathbb{Z}} \times \Sigma^{\mathbb{Z}}$
Since $\bar{x} = a_0 ... a_{j-1}$. Then the finite set $\sum_{t=0}^{j} i_t + \{0, 1\} \subset N(W, E)$

Since a_j, b_j are arbitrary values and the head moves with the tape by our rule 1. When the number of steps goes to the sum without addition, the corollary 3 is true, since the process describe the orbit of the one sided full shift, which is chaotic, according to [4], hence sensitive. The other element is in $N(W, E)$, since the configuration in general is separated by the elements of Q.

Extension of the result for infinitely many different elements

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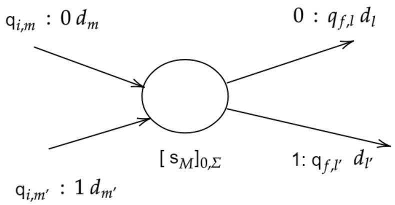
Firstly, fix $j \in \mathbb{N}$ as the first arrival to $x_j \neq y_j$. Since each relation between states have the same input and output symbols



In this last picture, m, m', l, l', M are all arbitrary. Now, suppose that the orbit of a cylinder of length j is well known for each process and Q have M states. Then, since Proposition 2 is true, it's possible to find an infinite sequence of integers, whose has a maximum, which be an element of W. For this occasion, we will assume that j is large enough and for a sequence $(k_r)_{r=1}^{\infty}$ with $x_{k_r} \neq y_{k_r}$ and $k_r \in [[1, j]]$. Since the rule 1 is assumed, I can construct a set $n_r = j + \sum_{r'=1}^{r} k_{r'}$, when $r' \neq 0$ and j otherwise. Since j is quite big, I have the following remark:

Proposition 5

Let $x, y \in \Sigma^{\mathbb{Z}}$ and the system has rule 1, then
$\left\{ \sum_{i=1}^{n(r)} l_i + \{0, ..., r+1\} \right\}_{r=0}^{\infty} \subset N(W, E)$

Question 1

Now, by *proposition 5* , we can affirm that since the system $(\Sigma^{\mathbb{N}}, \sigma)$ is σ-sensitive, then the system is δ-sensitive, if the first $j-1$ elements of the tape are all equal. Now, in general, we don't know if both machines has simultaneous arrivals in I_i, reason why we consider in particular the *rule 1*.

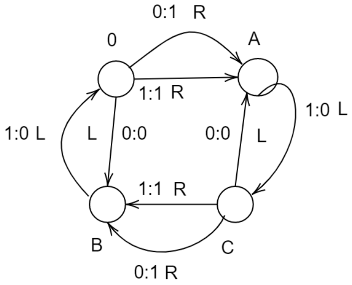
With the result of Proposition 4, we can affirm the following:

Proposition 6

The system with rule 1 is $b\mathcal{F}_{inf}$

This result is helpful for research other type of dynamical properties and classify stronger levels of chaoticity for machines known (ex. The aforementioned BINSMART is minimal and also sensitive, since always there's a disturbance on the side of the tape to be scanned will be seen at the head). Since this rule can be inherited depending the information that one has about the cylinders or their construction, this system can be described as sensitive in a global sense.

About the machine



Suppose that $x \in [11111111.11111100000]_{-9}$ and
$y \in [11111111.1111100001]_{-9}$
Here, fixing $j = 6$, the number of steps, the number of steps are 11, since $x_0 = 1$ and $y_0 = 1$. Since from $j > 6$, the elements are different, it's easy to check that the system is sensitive, but not chaotic. Also, this machine is almost periodic in both sequences.

Remark

Proposition 7

Let $(\mathcal{M}_\omega, \delta)$ be a system with rule 1. Suppose in addition that there is a $r' > 1$, such that $k_{r'} > 1$, then the system is chaotic in sense of Li-Yorke.

Since the machine can return back to the left side and project infinitely many elements equal and different, since for any process, it contains a configuration generated by a dynamic with σ-sensitive.
Now, we just need to check that having any two different process,we just need to check if we can write any sub process as uncountable. And that's our point:

Question 2

Proposition 8

Let $(\mathcal{M}_\omega, \delta)$ be a system with rule 1. There exists a semi-conjugated map $\pi : X \to \Sigma^{\mathbb{N}_0}$, such that if X is perfect and $\dim(X) < \infty$, then the system is chaotic in sense of Li-Yorke.

Future works

- Find Turing Machine that are chaotic in sense of ergodic measures
- Find a Turing machine, which would be locally chaotic in other sense, or it has no rule 1.

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